Name: Feedback

SA402 · Dynamic and Stochastic Models Fall 2023 · Uhan

Quiz 4 – 9/20/2023

Instructions. You have 15 minutes to complete this quiz. You may use your plebe-issue TI-36X Pro calculator. You may not use any other materials.

Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

Problem	Weight	Score
1a	1	
1b	1	
1c	1	
2	1	
Total		/ 40

Problem 1. Consider the Markov chain defined by the following one-step transition probability matrix:

$$\mathbf{P} = \begin{bmatrix} 0.1 & 0.2 & 0.4 & 0.1 & 0.2 \\ 0 & 0.6 & 0.4 & 0 & 0 \\ 0 & 0.9 & 0.1 & 0 & 0 \\ 0.5 & 0.2 & 0.2 & 0 & 0.1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

There are two recurrent classes: $\mathcal{R}_1 = \{2, 3\}$ and $\mathcal{R}_2 = \{5\}$.

a. Classify each of the 5 states as transient or recurrent. No explanation necessary.

Most of you had the right idea here.

Note that the problem gives you the two recurrent classes in this Markov chain. See the bottom of page 3 in Lesson 6 for guidance on how to use this information to classify the states as transient or recurrent.

b. Suppose the Markov chain starts in state 4. What is the probability that the Markov chain is absorbed into state 5?

See Example 6 in Lesson 6, as well as Problem 3 in the Lesson 6 Exercises, for similar examples.

Note that you need to identify the set of all of the transient states $\mathcal{T}$ in order to compute $\alpha_{\mathcal{T}\mathcal{R}}$ correctly.

Here is the one-step transition probability matrix from the previous page, for your convenience:

$$\mathbf{P} = \begin{bmatrix} 0.1 & 0.2 & 0.4 & 0.1 & 0.2 \\ 0 & 0.6 & 0.4 & 0 & 0 \\ 0 & 0.9 & 0.1 & 0 & 0 \\ 0.5 & 0.2 & 0.2 & 0 & 0.1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

c. Suppose the Markov chain has reached state 2. What is the steady-state probability of being in state 3?

See Example 5 in Lesson 6, as well as Problems 4, 5, and 6c in the Lesson 6 Exercises.

Note that the problem gives you the two recurrent classes in this Markov chain.

Problem 2. Consider a model of an elevator's movement from floor to floor in a high-rise building, in which the state of the system is defined as the floor on which the elevator is currently stopped, and the time step is defined to be the number of stops. Describe what assumptions need to be made in order for the Markov property to hold. (You do not need to discuss whether these assumptions are realistic.)

Many of you described the assumptions needed for both the Markov property and the time stationarity property to hold, but the problem only asked about the Markov property.

Recall that the Markov property says that the next state visited only depends on the last state visited (see page 1 of Lesson 7).

See Example 1 in Lesson 7 and Problem 1 in the Lesson 7 Exercises for similar examples.